Personal Solution Manual for
"Introduction to Physical Hydrology"
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Chapter 1

Introduction

Note to the reader, in this text the χ character is often used to denote a specific location x at which a phenomenon takes place. This is simply for the sake of brevity. If the character has any other meaning this will be explicitly mentioned.

Exercise 1.1

	Quantity	A	B	C
a	Interception evaporation	-	+	++
b	Interception storage	-	++	+
c	Stemflow	0	++	0
d	Throughfall	-	++	-
e	Infiltration	-	++	-
f	Soil evaporation	0	+	++

Exercise 1.2

$P + SW_{in} + GW_{in} = E_a + SWout + GW_{out} + WD + (\Delta S/\Delta t)$

Exercise 1.3

a Ice to water volume ratio is 0.9:1. $1mi^3 = 4.168km^3$ which leaves us with $2565932km^3 = 2.566 * 10^15m^3$

b $5.985 * 10^11m^3/yr = 598.5km^3/yr$

c $t = \frac{2.566 * 10^15}{5.985 * 10^11} = 4287 years$

Exercise 1.4

$1ft = 0.3048m, 1ft^3 = 0.0283m^3$ therefore $147m^3$.

Exercise 1.4.1

a Assuming no change in storage the water balance stands as follows: $\bar{P} = \bar{Q} + \bar{G} + \bar{E}$

b $500mm/yr$ which equals $37.5 * 10^8 m^3/yr$.

Exercise 1.4.2

$P = W + \frac{\Delta S}{\Delta t}$. After unit conversion we are left with P=20mm/40min and Q=15mm/40min. The difference is the soil/surface storage. The depth times the area yields $\Delta S = 50m^3$.

Exercise 1.5

$P = Q + E_a + (\Delta S/\Delta t)$ with no change in storage. $175 * 10^6 m^3/yr = 291.7mm/yr$.

Chapter 2

Atmospheric Water

2.1 Exercise 2.1

a $0.1kPa = 1hPa = 1mbar$. At $20°C$ the saturation vapour pressure is 23.4mbar which equals 2.34kPa.

b $0.6 * 2.34kPa = 1.404kPa$.

c When dew starts at the dew point the air is fully saturated $e_a \rightarrow e_s$. For the given volume this occurs at $T = 12.04°C$.

Exercise 2.2

$1atm = 1013.25N/100cm^2$. Using $F = ma$ we find $m = 103.29kg$.

Exercise 2.3

The south-east

Exercise 2.4

R_n	H	λE_a	G
+	+	+	-
-	-	0	-

Exercise 2.5

a $0.1inch/day$ evaporated from the pan which equals $2.48mm/day$.

b $0.7 * 2.48 = 1.7mm/day$.

Chapter 3

Groundwater

Exercise 3.7.1

a The flow is rightward parallel to the tube walls.

b $i = \frac{h_2 - h_1}{\Delta s} = \frac{100 - 120}{50} = -0,4,\ n_e = 0,4,\ K = 10m/day,\ A = 600cm^2 = 0,06m^2$. Therefore $Q = -iKA = 0,24m^3/s$

c $q = -iK = 4m/day,\ v_e = \frac{q}{n_e} = 10m/day =,\ \Delta s = \sqrt{l^2 + (z_2 - z_1)^2} = 50cm,\ t = 0,5/1,157 * 10^{-4} = 4320s = 1hr + 12min.$

Exercise 3.7.2

$q = |Ki| = 5 * \frac{1}{1000}.\ v_e = \frac{0,005}{0,4} = 0,0125m/day.\ t = \frac{456,25}{0,0125} = 10yrs.$

Exercise 3.7.3

$K = 86400\kappa\frac{\rho}{\mu}g.\ K_{5^\circ C} = 0,28m/day.\ K_{60^\circ C} = 0,88m/day.$

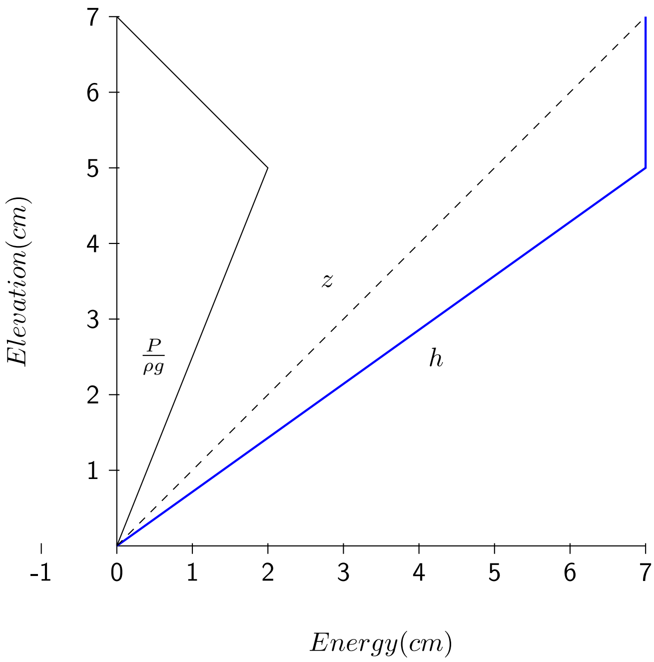
Exercise B3.2

$\frac{Q}{A} = 558,159mm.$

Exercise 3.7.4

$Q = -KA\frac{\Delta h}{L}$ and $Q = \frac{L + \frac{P_1}{\rho g}}{L}$. From this we can calculate the respective hydraulic conductivities as: $K_1 = 0,0625m/day$ and $K_2 = 0,0179m/day$.

Exercise 3.7.5



b Taking the bottom of the sample as z=0 then h=0 cm.

c Since there is no net flow over the water body above the sample $h_{surface} = h_{top} = 7cm$.

d $i = \frac{h_{top} - h_{z=0}}{L} = \frac{7-0}{5} = 1,4$

e $K = \frac{Q}{iA} = \frac{0,01344}{2,8 * 10^{-3}} = 4,8m/day$

f Sandy loam, it has a much to high hydraulic conductivity for clay.

Exercise 3.9

a The confined flow equation applies, $h(x) = C_1x + C_2$. Filling in the boundary conditions we find $C_1 = \frac{1}{100}$ and $C_2 = 10$. Therefore: $h_{25} = 9,75m$, $h_{50} = 9,50m$ and $h_{75} = 9,25m$.

b $Q' = KD\frac{dh}{dx} = 5 * 8 * \frac{1}{100} = 0,4m^2/day$

c $v_e = \frac{Q'/D}{n_e} = 0,15m/day$

d $667days$

Exercise 3.10.2

a $h(x) = C_1 x + C_2$. applies here.

x (in m)	h (in m)
25	9,75
50	9,5
75	9,25

b $K_{tot} = \frac{K_1 D_1 + K_2 D_2}{D} = \frac{60+10}{8} = 8,75 m/day$. $Q' = K_{tot} D i = 8,75{*}8{*}\frac{1}{100} = 0,7 m^2/day$. With the first layer being the upper layer and the second layer the bottom one.

c $q_1 = K_1 i = 0,1 m/day$ and $q_2 = K_2 i = 0,05 m/day$.

d $v_{e-1} = \frac{q_1}{n_{e-1}} = 0,1/0,4 = 0,25 m/day$ and $v_{e-1} = \frac{q_2}{n_{e-2}} = 0,05/0,1 = 0,5 m/day$.

e $t_1 = 100/0,25 = 400 days$ or $t_2 = 100/0,5 = 200 days$.

Exercise 3.10.3

As the layers are equally thick the formula's simplify. $K_{tot} = (K_1 + K_2 + K_3 + K_4)/4 = (1+5+10+50)/4 = 16,5 m/day$ and $k = \frac{4}{1/k_1 + 1/k_2 + 1/k_3 + 1/k_4} = 3,030 m/day$

Exercise 3.10.4

a $i = \frac{\Delta h}{d} = -0,0375$ and $k = \frac{d}{c} = 0,0088888...$ Therefore $q = 1/3000 m^2/day$ and $v_e = 1/1000 m/day$. $t = 8000 days$.

b $i = \frac{2\Delta h}{d} = -0,0375 * 2$ and $k = 0,0088888...$ Therefore $q = 2/3000 m^2/day$ and $v_e = 2/1000 m/day$. $t = 2000 days$.

c v_e doubles and t decreases by 75%.

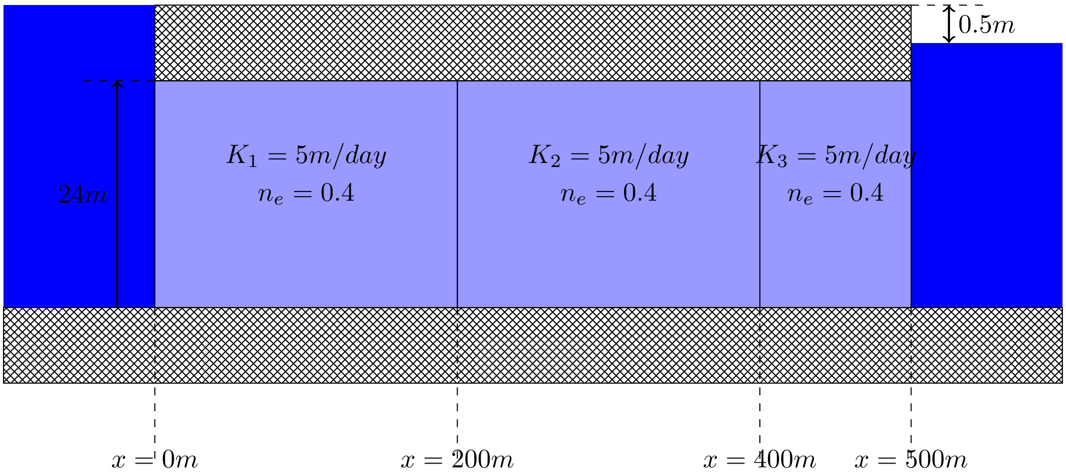
Exercise 3.10.5

a $c = \frac{d_1}{k_1} + \frac{d_2}{k_2} = 2,5/5 * 10^{-3} + 10/10^{-2} = 1500 days$.

b Because there is no change in storage the continuity equation applies and -ki over the first layer is the same as -ki over the second layer. Writing this out we find the following statement: $-5{*}10^{-3}\frac{h_{2,5} - 18,1}{2,5 - 0} = -10^{-2}\frac{17,5 - h_{2,5}}{12,5 - 2,5}$ from which $h_{2,5}$ can easily be isolated as $h_{2,5} = 17,9m$.

c $v_{e-1} = 4 * 10^{-3} m/day$, $v_{e-2} = 2 * 10^{-3} m/day$, $t = 5625 days$.

Exercise 3.10.6



a Using the same formula as the one for the upward seep in figure 3.11 we can say that: $q_1 = q_2 = q_3$ and therefore $c_1 = c_2 = c_3$. To determine the hydraulic conductivity we can substitute the latter as: $\frac{L}{K} = \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}$. Filling in the given values we get $K = 6.25m/day$. $Q' = -KDi = 0.15m^2/day$.

b Since the effective porosity n_e is constant for all compartments we may say $v_e = \frac{q}{n_e} = 0.015625m/day$ and therefore $\Delta t = \frac{L}{v_e} = 32000days$.

c Since $q_1 = q_2 = q_3$ we must have $-K_1 i_1 = -K_2 i_2 = -K_3 i_3$.

$K(inm/day)$	i
5	$-1.25 * 10^-3$
10	$-0.625 * 10^-3$
5	$-1.25 * 10^-3$

d From c it can readily be seen that i and K are inversely proportional as 2:1:2.

Exercise 3.11.1

a From the piezometric heads at A, B and C we can derive two hydraulic gradients between A and B and between B and C. $i_{AB} = -1,5 * 10^{-4}$ and $i_{BC} = -2 * 10^{-4}$. These can be combined using the Pythagorean Theorem to get the resultant gradient parallel to the flow direction which is $i_{flow} = -2,5 * 10^{-4}$.

b The angle between —AB— and i_{flow} can be determined by taking $cos\theta = \frac{\vec{BA} \cdot \vec{i_{flow}}}{|BA||i_{flow}|}$. From which we find $\alpha = 53°$.

Exercise 3.11.2

a $P + Q_{seepage} = E_a + Q_{pump}$. The actual evaporation $E_a = ((2/5)*600) + ((3/5)*420)mm/yr = 492mm/yr$.

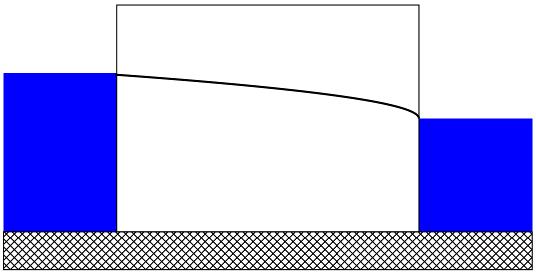
b $P = 750mm/yr$. $E_a = 492mm/yr$. There is no change in storage. $Q_{pump} = \frac{2*10^6 m^3/yr}{5*10^6 m^2} = 0,4m/yr = 400mm/yr$. Therefore using the water balance $Q_{seep} = 492+400-750 = 142mm/yr = 0,4mm/day$.

c $(\frac{\Delta S}{\Delta t})_{surfwater} = 200mm/yr$. Also $(\frac{\Delta S}{\Delta t})_{watertable} = 80mm/yr$. $(\frac{\Delta S}{\Delta t})_{total} = ((2/5)*200) + ((3/5)*80) = 128mm/yr$. Also taking actual evaporation from a and Q_{pump} from b we can determine using the water balance that $Q_{seep} = 492+400+128-750 = 270mm/yr = 0,7mm/day$.

Exercise 3.12

$T = \frac{Q_B - Q_A}{2Li} = \frac{|Q_r| + |Q_l|}{2Li} = KD = 4320m^3/day$.

Exercise 3.15.1.1



a Using $h = \sqrt{\frac{h_L^2 - h_0^2}{L}x + h_0^2}$ we obtain:

x (in m)	h (in m)
40	5,53
80	5,02
120	4,45
160	3,79

c Using the Dupuit-Forchheimer equation we obtain the volume flux Q'. $Q' = -\frac{1}{2}K\frac{h_L^2 - h_0^2}{L} = -0,5\frac{9-36}{200} = 0,0675m^2/day$.

Exercise 3.15.1.2

The water level in the left canal is given by $h_0 = \sqrt{\frac{6,25^2 - 6,75^2}{65}x - 10 + 6,75^2} = 6,82m$. The water level in the right canal can be determined using the unconfined flow equation, $h^2(x) = C_1x + C_2$, filling in the given piezometer heads to declare two formula's and then combining them as a system of equations as follows:

$6,75^2 - C_1 * 10 = C_2$

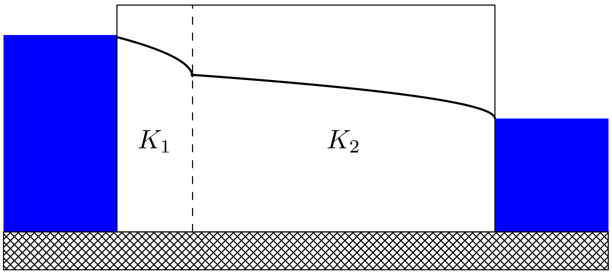
$6,25^2 = C_1 * 75 + 6,75^2 - C_1 * 10$

$\frac{6,25^2 - 6,75^2}{65} = C_1 = -0,1$

From this follows: $C_2 = 46,563$

$h_{100} = 6,05m$

Exercise 3.15.1.3



a For the left compartment: $h^2(x) = C_1 x + C_2$. Filling in the boundary conditions: $x = 0 \rightarrow C_2 = 36$. $x = 40 \rightarrow C_1 = \frac{h_{40}^2 - 36}{40}$.
Therefore $h^2 = \frac{h_{40}^2 - 36}{40} x + 36$.
For the right compartment: $h^2 = C_3 x + C_4$. Filling in the boundary conditions: $x = 40 \rightarrow C_4 = h_{40}^2 - 40C_3$. $x = 200 \rightarrow C_3 = \frac{9 - h_{40}^2}{160}$.
Therefore $h^2 = \frac{9 - h_{40}^2}{160} x + h_{40}^2 - 40\frac{9 - h_{40}^2}{160}$.
$Q'_{left} = Q'_{right} \rightarrow -K_l(h\frac{dh}{dx})_{left} = -K_r(h\frac{dh}{dx})_{right}$
With $K_l = 1m/day$ and $K_r = 10m/day$. If we now determine the derivatives of h and reorder the terms in the continuity equation above we can find a value for h_{40}.

$$h'_l(x) = \frac{1}{2}\frac{1}{\sqrt{\frac{h_{40}^2 - 36}{40}x + 36}}\frac{h_{40}^2 - 36}{40}$$

$$h'_r(x) = \frac{1}{2}\frac{1}{\sqrt{\frac{9 - h_{40}^2}{160}x + h_{40}^2 - 40\frac{9 - h_{40}^2}{160}}}\frac{9 - h_{40}^2}{160}$$

These can be combined as stated previously:

$$-\frac{1}{2}\frac{1}{\sqrt{\frac{h_{40}^2 - 36}{40}x + 36}}\frac{h_{40}^2 - 36}{40} = -10\frac{1}{2}\frac{1}{\sqrt{\frac{9 - h_{40}^2}{160}x + h_{40}^2 - 40\frac{9 - h_{40}^2}{160}}}\frac{9 - h_{40}^2}{160}$$

$$-\frac{1}{2}\sqrt{\frac{9 - h_{40}^2}{40} + h_{40}^2 - \frac{9 - h_{40}^2}{40}}\frac{h_{40}^2 - 36}{40} = -10\frac{1}{2}\sqrt{\frac{h_{40}^2 - 36}{40}40 + 36}\frac{9 - h_{40}^2}{160}$$

$$-\frac{1}{2}h_{40}\frac{h_{40}^2 - 36}{40} = -10\frac{1}{2}h_{40}\frac{9 - h_{40}^2}{160}$$

$$\frac{h_{40}^2 - 36}{80} = \frac{9 - h_{40}^2}{32}$$

$h_{40}^2 = 16,71429$
$h_{40} = 4,088m$.

c Since there is no change in storage within the two compartments the continuity equation applies. Therefore we only have to calculate the flow through one compartment. The Dupuit-Forchheimer equation can be used to calculate the flow:
$Q' = -\frac{1}{2}K\frac{h_L^2 - h_0^2}{L}$ with $h_L = h_{40}$ and L being the width of the first compartment. Filling out the terms yields: $Q' = 0,241m^2/day$.

d Yes, it does change. The degree to which it changes can be derived from the proof in paragraph a from the forth last line as follows: $K_l\frac{h_{40}^2 - 36}{80} = K_r\frac{9 - h_{40}^2}{320}$
$320K_l(h_{40}^2 - 36) = 80K_r(9 - h_{40}^2)$

$320K_l h_{40}^2 - 320K_l * 36 = 80K_r * 9 - 80K_r h_{40}^2$
$320K_l h_{40}^2 + 80K_r h_{40}^2 = 80K_r * 9 + 320K_l * 36$
$h_{40} = \sqrt{\frac{80K_r * 9 + 320K_l * 3}{320K_l + 80K_r}}$
$h_{40} = \sqrt{\frac{9K_r + 144K_l}{4K_l + K_r}}$
The lower the conductivity the steeper the hydraulic gradient becomes.

Exercise 3.15.1.4

a For the left compartment the confined flow equation applies: $h(x) = C_1 x + C_2$. For the right compartment the unconfined flow equation applies: $h^2(x) = C_1 x + C_2$. In this exercise subscripts 3 & 4 will be used for the right compartment instead of 1 & 2 to prevent confusion. Filling in the boundary conditions for the left compartment yields: $C_2 = 10m$ as $x = 0$. C_1 can then be solved for as: $C_1 = \frac{-2}{\chi}$ with χ being the distance x from the origin where the left compartment is separated from the right compartment by the transition from confined to unconfined flow. With both constants of the confined flow equation known it takes the following shape:

$$h_\chi(x) = \frac{-2}{\chi}x + 10 \tag{3.1}$$

The same can be done for the right compartment. $C_4 = 64 - \chi C_3$ as h equals 8 at the left boundary. Similarly $C_3 = \frac{36 - C_4}{100}$. By substituting the former in the latter as follows:
$C_3 = \frac{36 - C_4}{100}$
$C_3 = \frac{36 - 64 + \chi C_3}{100}$
$100C_3 = -28 + \chi C_3$
$C_3 = \frac{-28}{100 - \chi}$
Then by filling in the C_3 and C_4 in the unconfined flow formula we obtain:

$$h_\chi^2(x) = \frac{-28}{100 - \chi}x + 64 - \frac{-28\chi}{100 - \chi} \tag{3.2}$$

The previous two equations for the hydraulic heads in the compartments can now be linked to each other through the continuity equation.

$$KD(\frac{dh}{dx})_{(left)} = K(h\frac{dh}{dx})_{(right)} \tag{3.3}$$

Filling in this equation with the derivatives of the in this section aforementioned equations for the respective hydraulic heads yields an equation solvable for χ. Note that on the right hand side of the continuity equation the central term of the derivative and h cancel out. This yields: $\frac{-14}{100 - \chi} = \frac{-16}{\chi}$. Therefore $\chi = 53,333$

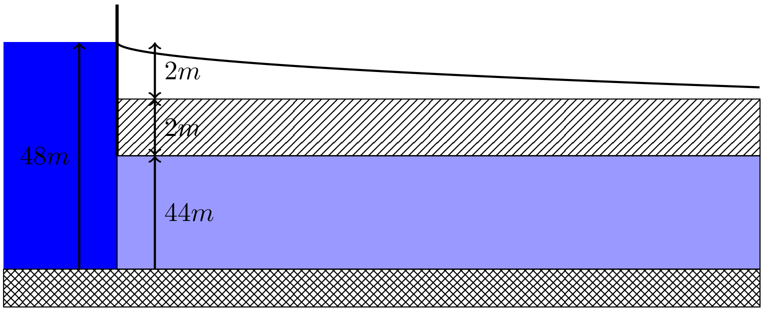
b From the continuity equation can be seen that the location of the change from confined to unconfined is placed irrespective of K. With Q' given as 1, $K = 3,33$.

Exercise 3.15.2.1

a

x	$h(x) inm$	$q_z(x) inmm$
10	43.98	1.98
100	43.77	1.77
250	43.46	1.46
500	43.07	1.07
1000	42.57	0.57
1500	42.31	0.31

b



c See paragraph a

d Horizontal Solution

$h(x) = h_a + C_1 e^{x/\lambda} + C_2 e^{-x/\lambda}$with $\lambda = \sqrt{KDc}$

$Q'_{x=0} = -KD(\frac{dh}{dx}_{x=0}); \; Q'_{x=L} = -KD(\,)$

$Q'_z = |Q'_{x=0}| + |Q'_{x=L}|$

$\frac{dh}{dx} \frac{C_1}{\lambda} e^{\frac{x}{-\lambda}} + \frac{C_2}{-\lambda} e^{\frac{-x}{\lambda}}$

$(\frac{dh}{dx})_{x=0} = \frac{C_1}{\lambda} e^0 + \frac{C_2}{-\lambda} e^0 = \frac{C_1 - C_2}{\lambda}$

$Q'_{x=0} = -KD\frac{C_1 - C_2}{\lambda}$

$(\frac{dh}{dx})_{x=L} = \frac{C_1}{\lambda} e^{\frac{L}{\lambda}} + \frac{C_2}{-\lambda} e^{\frac{-L}{\lambda}} = \frac{C_1}{e}^{\frac{L}{\lambda}} - C_2 e^{\frac{-L}{\lambda}} \lambda$

$Q'_{x=L} = -KD\frac{-C_1 e^{\frac{L}{\lambda}} + C_2 e^{\frac{L}{-\lambda}}}{\lambda}$

$|Q'_{x=L}| = -KD\frac{-C_1 e^{\frac{L}{\lambda}} + C_2 e^{\frac{L}{-\lambda}}}{\lambda}$

$Q'_z = |Q'_{x=0}| + |Q'_{x=L}| = -\frac{KD}{\lambda}(C_1 - C_1 e^{\frac{L}{\lambda}} - C_2 + C_2 e^{\frac{-L}{\lambda}})$

$\lambda^2 = KDc \Rightarrow \frac{\lambda}{c} = \frac{KD}{\lambda}$

$Q'_z = -\frac{\lambda}{c}(C_1 - C_1 e^{\frac{L}{\lambda}} - C_2 + C_2 e^{\frac{-L}{\lambda}}) = -\frac{KD}{\lambda}(C_1 - C_1 e^{\frac{L}{\lambda}} - C_2 + C_2 e^{\frac{-L}{\lambda}})$

Vertical Solution

$h(x) = h_a + C_1 e^{x/\lambda} + C_2 e^{-x/\lambda}$with $\lambda = \sqrt{KDc}$

$Q'_z \int_0^L q_z dx$

$q_z = -d\frac{h_a - h}{d}dx$

$h_a - h = -C_1 e^{\frac{e}{\lambda}} - C_2 e^{\frac{-x}{\lambda}}$

$Q'_z = \int_0^L -k\frac{-C_1 e^{\frac{e}{\lambda}} - C_2 e^{\frac{-x}{\lambda}}}{d}dx = \frac{k}{d}\int_0^L C_1 e^{\frac{e}{\lambda}} - C_2 e^{\frac{-x}{\lambda}} dx = \frac{1}{c}\left[\lambda C_1 e^{\frac{x}{\lambda}} - \lambda C_2 e^{\frac{-x}{\lambda}}\right]_0^L$

$Q'_z = \frac{\lambda}{c}(-C_1 + C_1 e^{\frac{L}{\lambda}} + C_2 - C_2 e^{\frac{-L}{\lambda}})$

$Q'_z = -\frac{KD}{\lambda}(C_1 - C_1 e^{\frac{L}{\lambda}} - C_2 + C_2 e^{\frac{-L}{\lambda}})$

Filling in the length, time, conductivities and thicknesses in either of the two aforementioned methods yields: $Q_z = 584000 m^3/yr$

e $Q_z = 188 m^3/day = 68620 m^3/yr$

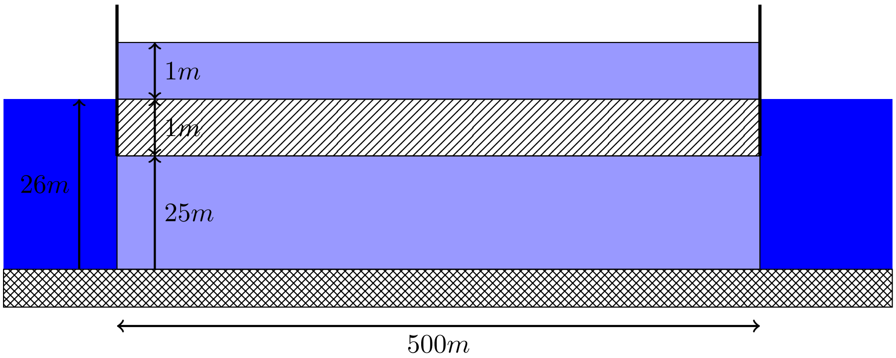
Exercise 3.15.2.2

a The leaky flow equation applies here: $h(x) = h_a + C_1 e^{x/\lambda} + C_2 e^{-x/\lambda}$. The leakage factor can be calculated from the given values as follows: $c = \frac{d}{k} = 1500 days$ and then $\lambda = \sqrt{KDc} = 600 meters$. Furthermore the boundary conditions given can be used to calculate C_1 and C_2. At $x = 0$ the leaky flow equation reduces to $16 = 13 + C_1 + C_2$. At the right boundary (at $x = 1200$) it reduces to $15 = 13 + C_1 e^2 + C_2 e^-2$. Solving the former for C_2 and substituting it in the last formula we get $2 = C_1 e^2 + (3 - C_1)e^-2$. Rearranging this for C_1 yields a value $C_1 = 0,22$. Therefore $C_2 = 2,78$ and $h(x) = 13 + 0,22 e^{\frac{x}{600}} + 2,78 e^{\frac{-x}{600}}$

b $Q'_z = \int_0^{1200} q_z dx$
$= \int_0^{1200} -\frac{k}{d}(h_a - h_x)dx$
$= \int_0^{1200} -\frac{k}{d}(13 + 0,22 e^{\frac{x}{600}} + 2,78 e^{\frac{-x}{600}})dx - \int_0^{1200} \frac{k}{d} 13 dx$
$= 1,524 m^2/day$

c The separation of flow sources is a region of zero flow. So $\frac{dh}{dx} = 0$. The derivative therefore is $\frac{0,22}{600} e^{\frac{x}{600}} + \frac{2,78}{-600} e^{\frac{-x}{600}} = 0$. Then taking the logarithm to solve for x:$\ln(0,22 e^{\frac{x}{600}}) = \ln(2,78 e^{\frac{-x}{600}})$, yields $x = 761 meters$.

Exercise 3.15.2.3



a The leaky flow equation, $h(x) = h_a + C_1 e^{x/\lambda} + C_2 e^{-x/\lambda}$, applies here. The vertical flow resistance c equals: $c = \frac{d}{k} = 1/0,002 = 500$. Therefore $\lambda = \sqrt{Tc} = 500$. Filling in the right boundary condition gives us $C_2 = -1 - C_1$. Then using the left boundary condition and substituting the value of C_2 in the leaky flow equation we get $C_1 = -0,2689$. Therefore $C_2 = -0,7311$.

b With the constants of the leaky flow equation known we can now derive the following results:

x	h
50	26,041
100	26,073
150	26,095
200	26,109
250	26,113

c
$$\begin{aligned} q_z &= -k\frac{h_a - h}{d} \\ &= -0,002\frac{27 - 26,113}{1} \\ &= -1,774 * 10^{-3} m/day \end{aligned}$$

d Method 1 - horizontal

$Q'_z = Q'_{x=0} = -KDi_{x=0}$

$i_{x=0} = (\frac{dh}{dx})_{x=0} = \frac{C_1}{\lambda} e^{\frac{x}{\lambda}} + \frac{C_2}{-\lambda} e^{\frac{-x}{\lambda}}$

want $h(x) = h_a + C_1 e^{x/\lambda} + C_2 e^{-x/\lambda}$

$\frac{dh}{dx}(0) = \frac{C_1 - C_2}{\lambda}$

$Q'_{x=0} = -\frac{500}{500} * (-0,2689 + 0,7311) = -0,4622 m^2/day$

At the right boundary $i_{x=500} = 9,244 * 10^{-4}$ and $Q'_z = 0,4622 m^2/day$

$Q'_{z,total} = 0,9244 m^2/day$

Method 2 - vertical $Q'_z = \int_0^{500} q_z dx$

$q_z = -\frac{k}{d}(h_a - h)$

$h_a - h = -C_1 e^{\frac{x}{\lambda}} - C_2 e^{\frac{-x}{\lambda}}$

$Q'_z = \frac{k}{d} \int_0^{500} C_1 e^{\frac{x}{\lambda}} + C_2 e^{\frac{-x}{\lambda}} dx$

$Q'_z = (\lambda C_1 e^{\frac{x}{\lambda}} - \lambda C_2 e^{\frac{-x}{\lambda}})\Big|_0^L * \frac{k}{d}$

$Q'_z = 0,9244 m^2/day$

e Pumping with precipitation should equal leakage. $Q'_{zP} = P * l = 0,5 m^2/day$. $Q'_{z-pump} = 0,4244 m^2/day$ must be transferred by pumping.

f Choosing the effective as 0,3 we get a velocity of $v = \frac{q_z}{n_e} = \frac{-1,774 * 10^{-3}}{0,3} = -5.9133 * 10^{-3} m/day$

Exercise 3.15.2.4

a This exercise is in essence the same as the previous one. $\lambda = \sqrt{240 * c}$ with $c = \frac{d}{k} = \frac{3}{0,002}$. Therefore $\lambda = 600$. If we use these values and apply them to the leaky flow equation along with the boundary conditions we get expressions for $C_1(C_2)$ and vice versa. Using this to substitute either C_1 or C_2 we get $C_1 = -0,3764$ and $C_2 = -1,6254. h(0) = 11,998 \approx 12 metres. h(1200) = 10,999 \approx 11 metres.$

b Both horizontally and vertically the seepage can be expressed as:

$$Q'_z = -\frac{KD}{\lambda}(C_1 - C_1 e^{\frac{L}{\lambda}} - C_2 + C_2 e^{\frac{-L}{\lambda}}) \tag{3.4}$$

with $L = 1200$ and $\lambda = 600$ and with $T = 240$ this yields $Q'_z = -1,524 m^2/day$.

c The water divide lies at $\frac{dh}{dx} = 0$.

$h(x) = h_a + C_1 e^{x/\lambda} + C_2 e^{-x/\lambda}$

$\frac{dh}{dx} = \frac{C_1}{\lambda} e^{\frac{x}{\lambda}} + \frac{C_2}{-\lambda} e^{\frac{-x}{\lambda}} = 0$

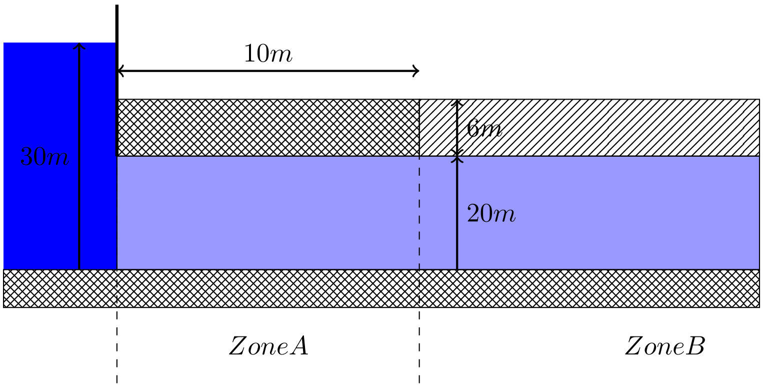
$\frac{C_1}{\lambda} e^{\frac{x}{\lambda}} = \frac{C_2}{\lambda} e^{\frac{-x}{\lambda}}$

$0,23157 e^{\frac{x}{\lambda}} = e^{\frac{-x}{\lambda}}$

$-1,4628 + \frac{x}{\lambda} = \frac{-x}{\lambda}$

$x = 0,73\lambda = 438,86 metres$

Exercise 3.15.2.5



a In zone A $h(x) = C_1x + C_2$applies. In zone B $h(x) = h_a + C_1e^{x/\lambda} + C_2e^{-x/\lambda}$applies. $C_2 = 30m$ and $C_1 = \frac{h_{10} - C_2}{10}$. Therefore $h_A = \frac{h_{10} - 30}{10}x + 30$

In zone B we can then say: $\lim_{x\to\infty} h_B \Rightarrow h(\infty) = 26 + C_3e^{\frac{\infty}{\lambda}} + C_4e^0$ and $C_3 = 0$. Using that and filling the left boundary condition for zone B we get: $h_B(10) = h_{10} = 26 + C_4e^{\frac{-10}{\lambda}}$. Therefore $C_4 = \frac{h_{10} - 26}{e^{\frac{-10}{\lambda}}}$ and $h_B = 26 + \frac{h_{10} - 26}{e^{\frac{-10}{\lambda}}}e^{\frac{-x}{\lambda}}$. h_{10} can be determined by equating the derivatives of the two formulas for the hydraulic head and calculating the only remaining variable h_{10} at x=10m. Simply equating the two formula's for the hydraulic heads themselves leaves a null statement $h_{10} = h_{10}$. We are allowed to do this because of the continuity equation of course.

$(\frac{dh}{dx})_A(10) = (\frac{dh}{dx})_B(10)$

$\frac{h_{10} - 30}{10} = \frac{h_{10} - 26}{e^{\frac{-10}{\lambda}}}\frac{-1}{\lambda}e^{\frac{-x}{\lambda}}$ with x=10m and $\lambda = \sqrt{KDc} = 200$

The power terms cancel out and leave us with:

$\frac{h_{10} - 30}{10} = \frac{26 - h_{10}}{\lambda}$

$h_{10} = \frac{6260}{210} = 29.810m$

b

x (in m)	h (in m)
5	29.826
15	29.562
25	29.389

c Horizontal solution.

$Q'_z = Q'_{x=0} = -KDi = -20 * 25 * (\frac{29.81 - 30}{10}) = 9.50m^2/day$

Vertical solution.

$Q'_z = \int_{10}^{\infty} q_z dx$

$q_z = \frac{-k}{d}(h_a - h)$

$h_a - h = -C_1e^{\frac{x}{\lambda}} - C_2e^{\frac{-x}{\lambda}}$ with $C_1 = 0$

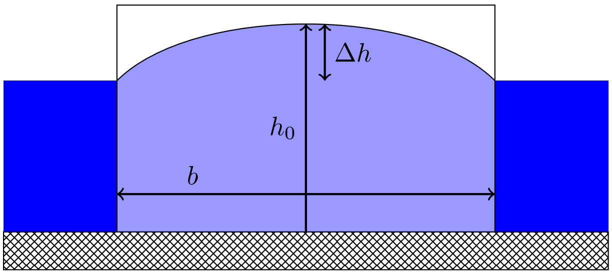
$Q'_z = (C_2\lambda e^{\frac{-x}{\lambda}})\Big|_{10}^{\infty} * \frac{-k}{d}$

$Q'_z = \frac{-k}{d}(C_2\lambda e^0 - C_2\lambda e^{\frac{-10}{\lambda}})$
Filling in all the known variables yields: $Q'_z = 9.52m^2/day$

d

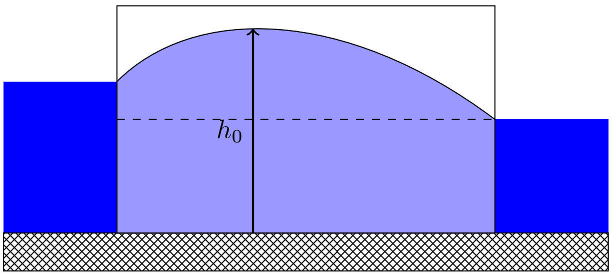
$x(inm)$	$q_z(inmm/day)$
5	0
15	46.5
25	44.1

Exercise 3.15.3.1



$T = KD = 50m^2/day$ and $\Delta h = 0,09metres$ and $N = 0,01metres$. By using the Hooghoudt equation we can solve for b, the distance between the two canals. $N = \frac{\Delta h}{(\frac{L^2}{2KD_{average}})}$. $0,01 = \frac{0,09}{\frac{L^2}{100}}$. $L = 30metres$ and therefore $b = 2L = 60metres$.

Exercise 3.15.3.2



The recharge equation for unequal water levels applies here: $h^2(x) = -\frac{N}{K}x^2 + C_1x + C_2$
Filling in the boundary conditions we get at $x = 0$ $h^2 = h_0^2 = C_2$. Then at $x = L$ we get $C_1 = \frac{h_L^2 - h_0^2}{L} + \frac{NL}{K}$.
The water divide lies at $\frac{dh}{dx} = 0$. So first we need to find the derivative of the recharge equation. This is done as follows:
$h^2(x) = -\frac{N}{K}x^2 + C_1x + C_2$
$\frac{dh^2}{dx} = \frac{-2N}{K}x + C_1 = 2h\frac{dh}{dx}$
$h\frac{dh}{dx} = \frac{-N}{K}x + \frac{C_1}{2}$
$h\frac{dh}{dx} = \frac{-N}{K}x + \frac{h_L^2 - h_0^2}{2L} + \frac{NL}{2K} = 0$
$\chi = \frac{K}{N}(\frac{h_L^2 - h_0^2}{2L} + \frac{NL}{2K})$

$\chi = \frac{K(h_L^2 - h_0^2)}{N2L} + \frac{L}{2}$

h_{max} lies at the water divide therefore: $h_{max} = h(\chi) = \sqrt{h_0^2 + \frac{(h_L^2 - h_0^2)\chi}{L} + \frac{N}{k}(L-\chi)\chi}$

Exercise 3.15.4.1

a This exercise assumes that the aquifer is isotropic, infinite, steady state, and has homogeneous flows. There is no correlation between the data points in the y-direction. However the data follows a negative linear formula in the x-direction. The formula is $h = 8,45 - \frac{1}{1000}x$. with $i = \frac{1}{1000}$ The $8metre$ and $7metre$ equipotential lines lie at $x = 450metres$ and $x = 1450metres$ respectively.

b $K = 10m/day$. $Q'' = -Ki = \frac{-10}{-100} = 1cm/day$

c $v = \frac{Q''}{n_e} = \frac{0,01}{0,4} = 2,5m/day$ in the x-direction.

d $y = 625metres$

e $Q = Q''WD = 0,001 * 250 * 20 = 50m^3/day$.

Exercise 3.15.4.2

a Following the continuity equation both are obviously the same as all water that flows into radius r_1 must flow through radius r_2 to be pumped out by the well.

b The flux densities follow the same reasoning. At any point in the aquifer the flux densities are equal if assessed perpendicular to the point source. It is the same argument as a but with equal sectors θ taken instead of a full circle.

Exercise 3.15.4.3

a $R = 1000metres$

b $\Delta h = 0,1m$

$h(x) = h_R + \frac{Q_0}{2\pi KD} ln\frac{r}{R}$ for $r_w \leq r \leq R$

$-0,1 = \frac{628}{2\pi 10 * 50} \ln \frac{r}{1000}$

$r = 606,38metres$

Exercise 3.15.4.4

a $h^2 = h_R^2 + \frac{Q_0}{\pi K} \ln \frac{r}{R}$

$h^2 - h_R^2 = \frac{Q_0}{\pi K} \ln \frac{r}{R}$

$-141 = \frac{Q_0}{\pi 20} \ln \frac{0,2}{5000}$
$Q_0 = 874,85 m^3/day$

b $h(x) = h_R + \frac{Q_0}{2\pi KD} ln \frac{r}{R}$ for $r_w \leq r \leq R$
$Q_0 = 930,69 m^3/day$

c There is a difference of $\frac{50}{47}$ as $2D = 50$ and $h + h_R = 47$.

Exercise 3.15.4.5

a As per the continuity equation we have: $|Q_0| = |Q_W|$. Therefore: $Q = Q_0 = WDKi$. The width can be determined with $W = \frac{Q_0}{Ti} = \frac{314}{10 * 50 * 0,001} = 628 metres$

b $r = \frac{-Q_0}{2\pi Ti} = \frac{-314}{2\pi 500 * 0,001} = -99,95 metres$
P(x;y)=(-99,95;0) P(x;y;z)=(-99,95;0;0-50), the stagnation point a line along the z-direction with length D.

Exercise 3.15.5.1

a $h(x) = h_R + \frac{Q_0}{2\pi KD} ln \frac{r}{R}$ for $r_w \leq r \leq R$
$h_1 = 10 - 0,26m$ with $\Delta h = 0,26m$
$h_2 = 10 - 0,15m$ with $\Delta h = 0,15m$
h_3 does not exist as $r > R$.
$h_{tot} = 9,597 metres$

b $h_1 = 10 - 0,26m$ with $\Delta h = 0,26m$
$h_{2new} = 10 - 0,15m$ with $\Delta h = -0,14m$
h_3 does not exist as $r > R$.
$h_{tot} = 9,89 metres$

Exercise 3.15.5.2

a $T_{tot} = T_1 + T_2$
$= D_1 K_1 + D_2 K_2 = 1100 m^2/day$

b $\Delta h = \frac{Q_0}{2\pi T} \ln \frac{r}{R}$. With all values known $\Delta h = -0,25 metre$.

c The lowering will decrease because the aquifer gets replenished from the canal.

d A mirror recharge well is used at distance r from the centre of the nature reserve. The mirror recharge well lies at 50 metres from the canal. It lies across from the real pumping well and the line connecting the two wells lies perpendicular to the canal. By superposing the two drawdowns the effect of the real well on the nature reservoir can be calculated.
$r = \sqrt{(400 + 2 * 50)^2 + 300^2} = 583,10 metres$

$\Delta h_{mirror} = \frac{Q_2}{2\pi T} \ln(\frac{r}{R}) = +0,22 metres$
$\Delta h_{tot} = -0,03 metres$
So yes the result corresponds fully with c as the drawdown decreases although it does not become zero.

e Since the layers are both fully saturated their fluxes depend solely on their ability to conduct water, the transmissivity T. $T_1 : T_2$ as 200 : 900 or as 2 : 9.

f Since $v = \frac{q}{n_e}$ and $n_{e1} = n_{e2}$ and $i_1 = i_2$ v only depends on K. Therefore $v_1 : v_2$ follows $K_1 : K_2$ which is 10 : 30 or 1 : 3.

Exercise 3.15.5.3

a $\Delta h = -0,1 metres,\ Q_0 = 314 m^3/day,\ r = 1000m.$ Therefore $\Delta h = \frac{Q_0}{2\pi T} \ln \frac{r}{R}$ is used. $R = \frac{r}{e^{\frac{2\pi T \Delta h}{Q_0}}} = 1105,23 metres.$ $\Delta h_{400m} = \frac{Q_0}{2\pi T} \ln \frac{400}{1105,23} = -1,016m$

b The drawdown at χ (at the barrier), on the pumping side is: $\Delta h_{\chi} = 2\Delta h_{400m} = \frac{Q_0}{\pi T} \ln \frac{400}{1105,23} = -2,03 metres.$

Chapter 4

Soil Water

Exercise 4.1.1

a 1 atm

b 1 atm

c 1 atm with $1atm = 10^5 Pa$

d 2 atm

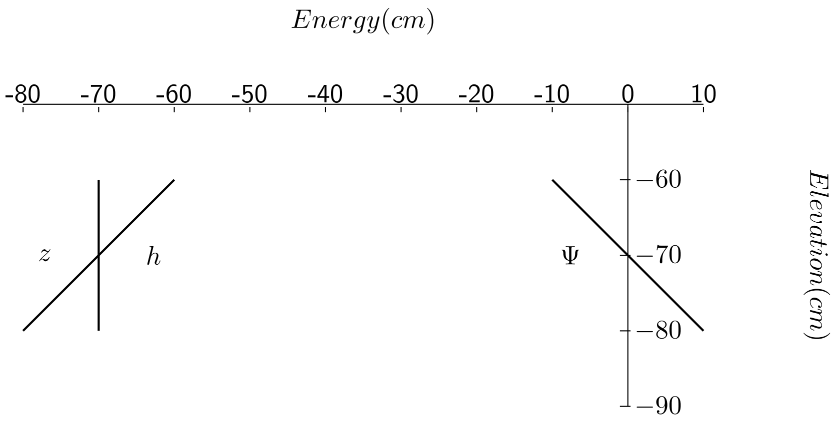
e 3 atm

f 1 cm

Exercise 4.1.2

$Pf = 2$ and therefore as $Pf = \log -\Psi$ the value of Ψ equals 1 metre. $\Delta h = z_{saturated} + \Psi = -3-1 = -4 metres$. $i = \frac{\Delta h}{\Delta z} = \frac{-4}{-2} = 2$. $K = 2{*}10^{-3} m/day$. $q = -Ki = -4{*}10^{-3} m^2/day$. $v = \frac{q}{n_e} = \frac{-4 * 10^{-3}}{0,5} = -8 * 10^{-3} m/day$. Time t after which all water on top of the semipermeable layer has infiltrated is $t = 250 days$.

Exercise 4.2



a $\Psi_{60} = z - \Psi_M - |\Delta z| = -60 + 90 - 20 = 10cm$
$\Psi_{80} = -80 + 90 - 20 = -10cm$
$|\Delta z|_{60} = 80cm$ and $|\Delta z|_{80} = 100cm$
$h_{60} = z + \Psi + |\Delta z| = -60 - 90 + 80 = -70cm$
$h_{80} = z + \Psi + |\Delta z| = -80 - 90 + 100 = -70cm$

b Depth of the watertable is at $\Psi = 0cm$ which is at $z = -70cm$.

Exercise 4.4.1

$\Delta\% * d = 15\% * 20cm = 3cm$
$\Delta\% * d = 10\% * 20cm = 2cm$
Adding these two together yields a sum of 5 cm.

Exercise 4.4.2

a

pF	Ψ	$\Theta_{\%A}$	$\Theta_{\%B}$
0	-10^0	42	57
1	-10^1	39	55
2	-10^2	12	51
3	-10^3	5	47
4,2	$-10^{4,2}$	2	35
5	-10^5	1,5	25
6	-10^6	1	9

b sand, clay

c 10% and 16%; $d = 40cm$.
$\Delta\% d_1 = 4cm$
$\Delta\% d_2 = 6,4cm$
Clay can provide more water than the sandy soil to the plants.

Exercise 4.8.1

a $K = 20mm/hr$, $S_f = -\Psi = 60mm$, $\Theta_i = 20\%$, $n_e = 45\% = \Theta_{sat}$, $i = \dfrac{z_f - h_0 + \Psi}{z_f}$, $h_0(z_0) = 20mm$, $h_f = z_f + \Psi$, $F = \Delta\Theta_{s-i} * z_f$

z_f(mm)	i	q (mm/hr)	F (mm)
-20	5	100	5
-40	3	60	10
-80	2	40	20
-160	1,5	30	40
-320	1,25	25	80
-640	1,125	22,5	160
-1280	1,0625	21,25	320

b As i decreases q decreases accordingly.

Exercise 4.8.2

$f_t - f_c = (f_0 - f_c)e^{-\alpha t}$, $f_c = 5mm/hr$

$$\frac{f_{t+\Delta t - f_c}}{f_t - f_c} = e^{-\alpha\Delta t}$$

$$\frac{25-5}{45-5} = e^{-\alpha 30} \quad \alpha = -\frac{\ln\frac{1}{2}}{30} = 0,0231$$

$$\frac{45-5}{\chi-5} = e^{-\alpha 30}$$

$\chi = 85 = f_0$, With these answers known we can use the Horton equation to produce the following table:

t	f_t
0	85
30	45
60	25
65	22,8
75	19,1
90	15
120	10

Exercise 4.8.3

Determine K as the asymptotic end value: $K = 0,4mm/min$. f is known op $t = 1$, therefore the following formula applies at that time:

$$f = \frac{1}{2}S_t^{-\frac{1}{2}} + K$$

With all values known the Sorptivity $S = 4,2mm/\sqrt{min}$.

Exercise 4.8.4

K is the slope between t=60 and t=30 so

$K = \dfrac{26-17}{60-30} = 0,3mm/min$, F=2,0 mm at t=1, therefore:

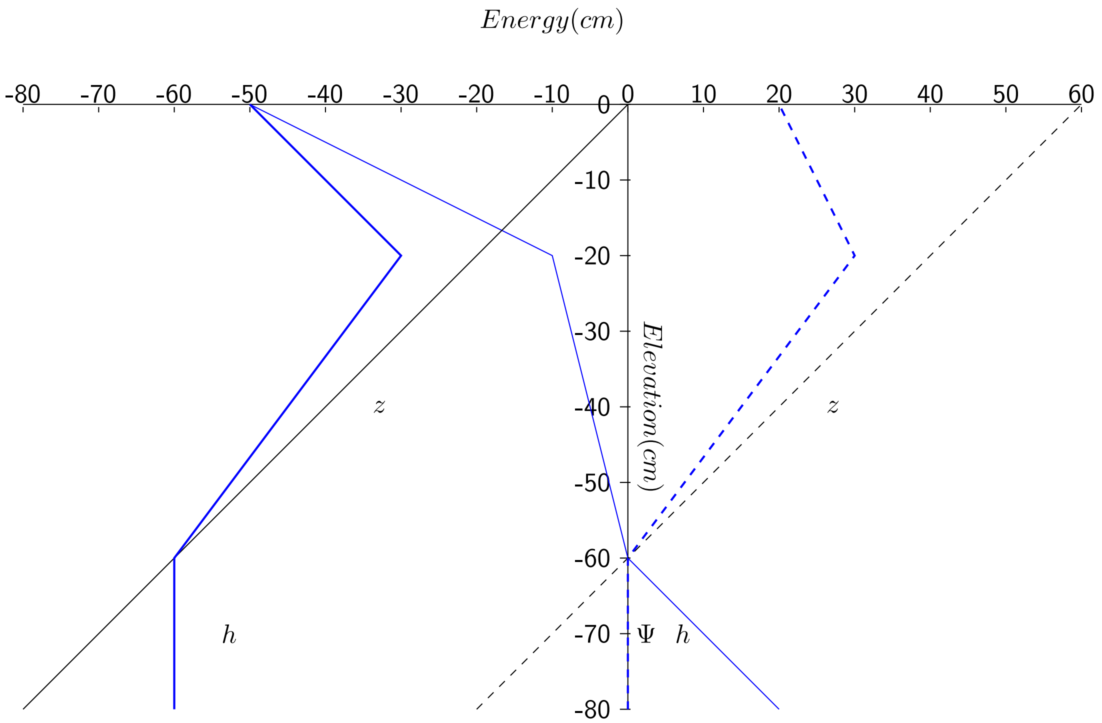
$S = F - K = 1,7mm/\sqrt{min}$

Exercise 4.8.5

$S = 20/\sqrt{2}cm/\sqrt{hr} = 200\sqrt{2}\frac{1}{\sqrt{60}}mm/\sqrt{min} = 36,51mm/\sqrt{min}$. $\Theta_i = 10\%$. $\Theta_s = 35\%$.

t(hr)	L(cm)
0	0
2	20
6	34,64
24	69,28

Exercise 4.8.6



b -60cm

c -20cm

d Hydrostatic and constant head $\rightarrow q = 0cm/day$.

e $i_e = \frac{\Delta h}{\Delta z} = \frac{-50 - -30}{-20} = 1$

f $\bar{\Psi} = \frac{-50 + -10}{2} = -30$
$K_e(\bar{\Psi}_e) = 248,6/30^{2,11} = 0,19cm/day$

g $q = i_e K_e = 0,19cm/day$

h $i_p = \frac{\Delta h}{\Delta z} = \frac{30}{40} = 0,75$. Therefore $i_p : i_e$ as $0,75 : 1$. But $K_p(\bar{\Psi}_p) = 248,6/5^{2,11} = 8,33 cm/day$. So we must conclude that $K_p(\bar{\Psi}_p) : K_e > i_p : i_e$

Exercise 4.8.7

a The continuity equation: $q_u = q_l \rightarrow -K_u i_u = -K_i i_i$. Therefore $\frac{K_u}{K_l} = \frac{3}{8} = \frac{i_l}{i_u}$

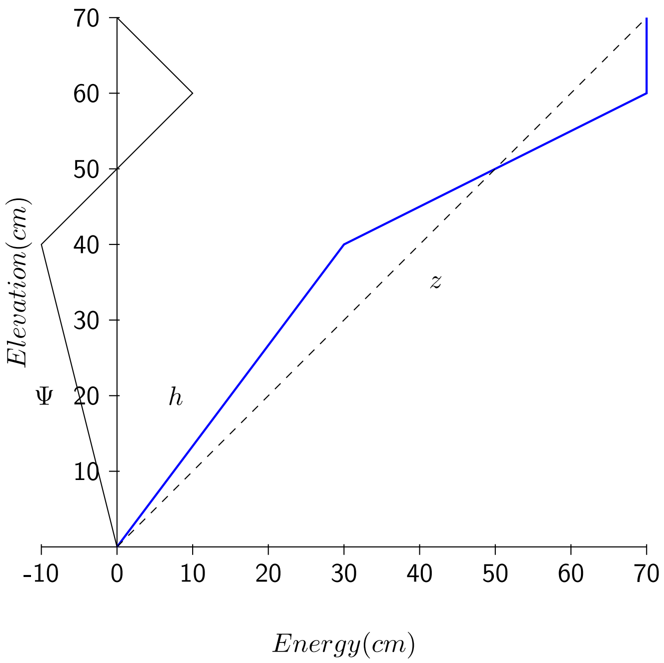
$h_{60} = z_{60} + \Psi_{60} = 60 + 10 = 70$ and $h_0 = z_0 + \Psi_0 = 0 + 0 = 0$

$i_u = \frac{h_{40} - h_{60}}{z_{40} - z_{60}} = \frac{h_{40} - 70}{-20}$

$i_l = \frac{h_0 - h_{40}}{z_0 - z_{40}} = \frac{h_{40}}{40}$

$\frac{i_l}{i_u} = \frac{3}{8} = \frac{(\frac{h_{40}}{40})}{(\frac{h_{40} - 70}{-20})}$

Now h_{40} is the only unknown and therefore $h_{40} = 30cm$, $i_u = \frac{30 - 70}{-20} = 2$ and $i_l = \frac{30}{40} = \frac{3}{4}$



b

1. The pressure head in the drainage pipe must be equal to zero.
2. The matric suction must be lower than the air-entry suction for both layers.
3. The continuity condition applies.

c $l = 40cm = \Delta z$, $K = 1,2mm/min$, $\Delta h = -30cm$, $i = \frac{-30}{40} = -0,75$ and therefore $q = -Ki = 0,9mm/min$.

Exercise 4.9

a The two flow domains have the following characteristics:

Matrix flow	Preferential flow
$f_r = 0,8$	$f_r = 0,2$
$n_e = 0,5$	$n_e = 0,4$
K=3mm/hr	K=100mm/hr
i=1	i=1

$q = 3mm/hr$, $t = 1hr$
$D = \frac{q}{n_e}$ as long as $q \not> = K = K * t$
$D_m = 6mm$ and $D_{pr} = 7,5mm$.

b $D_m = 6mm$ and

$$D_p r = \frac{q_{pr} + q_m(\frac{f_r - m}{f_r - pr})}{n_e} = \frac{6 + 3(\frac{0,8}{0,2})}{0,4} = 45mm$$

c Overland flow starts as both domains are draining at capacity. So when $K_{tot} = P+(P-3)*4$, a water depth P of 22,4 mm/hr. With $K_{tot} = 100mm/hr$.

d $\bar{n_e} = (0,8 * 0,5) + (0,2 * 0,4) = 0,48$
$\bar{K} = (0,8 * 3) + (0,2 * 100) = 22,4mm/hr$

e $\frac{P}{n_e} = \frac{3}{0,48} = 6,25mm$

f $\frac{P}{n_e} = \frac{6}{0,48} = 12,5mm$

g $\frac{P}{n_e} = \frac{22,4}{0,48} = 46,7mm$

h The maximum infiltration depth will be underestimated.

Chapter 5

Surface Water

Exercise 5.1.1

$H_C = 0,7m$, $w = 10m$, $Q = wH_C\sqrt{gH_C} = 10 * 0,7 * \sqrt{9,81 * 0,7} = 18,34m^3/s$.

Exercise 5.1.2

$\frac{V_1^2}{2g} + \frac{P_1}{\rho g} = \frac{V_2^2}{2g} + \frac{P_2}{\rho g}$
$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$
$P_1 - P_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$
$\Delta P_{1-2} = \frac{\rho}{2}(\frac{Q_2^2}{A_2^2} - \frac{Q_1^2}{A_1^2})$
$\Delta P_{1-2} = \frac{\rho}{2}(\frac{Q_2^2 A_2^2 - Q_1^2 A_2^2}{A_2^2 A_1^2})$
$Q_1 = Q_2$
$\frac{2\Delta P_{1-2}A_2^2A_1^2}{\rho} = Q^2A_1^2 - Q^2A_2^2$
$Q^2 = \frac{2\Delta P_{1-2}A_2^2A_1^2}{\rho(A_1^2 - A_2^2)}$
$Q = A_1A_2\sqrt{\frac{2\Delta P_{1-2}}{\rho(A_1^2 - A_2^2)}}$
QED. This can still be rewritten further to a form without the pressure term making the formula contain only directly measurable observables.
$Q = A_1A_2\sqrt{\frac{2g(h_1 - h_2)}{A_1^2 - A_2^2}}$

Exercise 5.1.3

$A = H_2^2 \tan 0,5\Theta$
$H_2 = \frac{2}{3}H_1$
$v_2 = \sqrt{\frac{2}{3}gH_1}$
$A = \frac{4}{9}H_1^2 \tan 0,5\Theta$

$Q = \sqrt{\frac{2}{3}gH_1} * \frac{4}{9}H_1^2 \tan 0,5\Theta$

$Q = \frac{4}{9}\sqrt{\frac{2}{3}}gH_1^{\frac{5}{2}} \tan 0,5\Theta$

$Q = CH_1^{\frac{5}{2}} \tan 0,5\Theta$

Exercise 5.2.1

$Q = \sum_{i=1}^{n}(\frac{\overline{V_{i-1}} + \overline{V_i}}{2})(\frac{H_{i-1} + H_i}{2})(w_i - w_{i-1})$

d	0cm	20cm	40cm	60cm	80cm
H	0	15	23	17	0
$v_{at0,8H}$	0	18	26	20	0
$v_{at0,4H}$	0	13	20	15	0
$v_{at0,2H}$	0	8	14	10	0
$v_{at0,0H}$	0	0	0	0	0
$\bar{v}$	0	12,7	19,2	15,5	0

$Q_{0-20} = 20((\frac{0 + 12,7}{2})(\frac{0 + 15}{2})) = 952,5cm^3/s$
$Q_{20-40} = 20((\frac{12,7 + 19,2}{2})(\frac{15 + 23}{2})) = 6061cm^3/s$
$Q_{40-60} = 20((\frac{19,2 + 15,5}{2})(\frac{23 + 17}{2})) = 6940cm^3/s$
$Q_{60-80} = 20((\frac{15,5 + 0}{2})(\frac{17 + 0}{2})) = 1317,5cm^3/s$
$Q_{tot} = \sum_{i=1}^{n}(Q_i) = 1527cm^3/s = 15,27L/s$

Exercise 5.2.3

$Q = Q_i\frac{C_\chi}{C_d} = Q_i\frac{1}{C_{rd}} = Q_i\frac{1}{C_r}$, with $C_r = 6,486 * 10^{-6}EC - 2,871 * 10^{-3}$, in which $EC = 765\mu S/cm$. Since $Q_i = 0,1L/s$ this yields $Q = 47,8L/s$

Exercise 5.2.4

$Q_i = 18,5L$. $Q_{river} = \frac{Q_i}{\int_{t_1}^{t_2}}(C_{rd} - C_{rb})dt = \frac{18,5}{0,405} = 45,68L/s$.

Exercise 5.2.5

$C_1Q_1 + C_2Q_2 = C_3(Q_1 + Q_2)$ and $Q_1 + Q_2 = Q_3 \rightarrow C_1(Q_3 - Q_2) + C_2Q_2 = C_3Q_3 \rightarrow Q_2 = \frac{C_3Q_3 - C_1Q_3}{C_2 - C_1} = 7,5L/s$. With all values now known $Q_1 = Q_3 - Q_2 = 10L/s$.

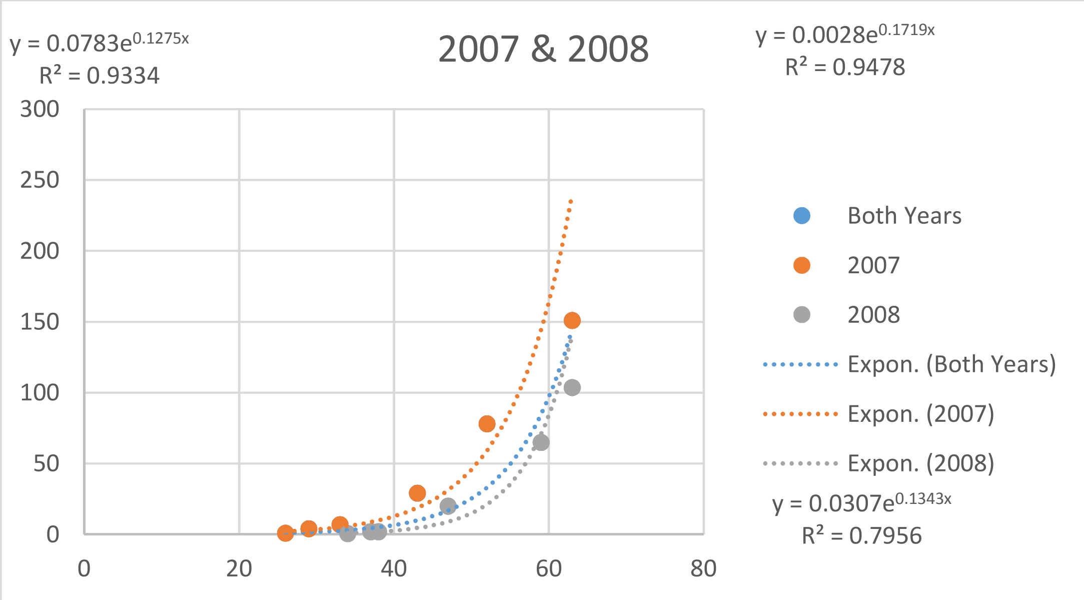
Exercise 5.2.6

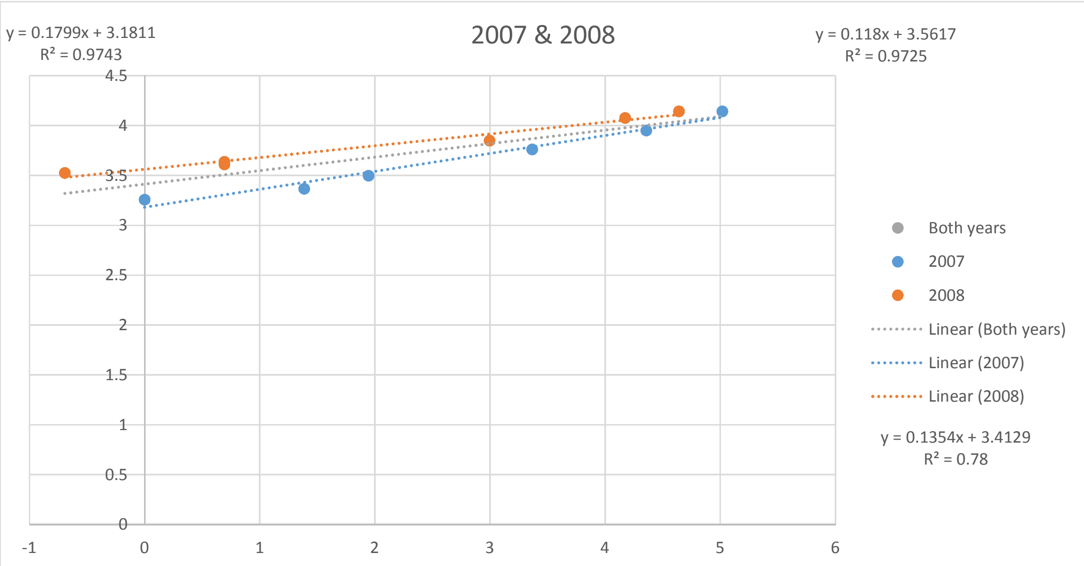
$C_1Q_1 + C_2Q_2 = C_3(Q_1 + Q_2) \rightarrow C_2Q_2 - C_3Q_2 = C_3Q_1 - C_1Q_1 \rightarrow Q_2 = \frac{C_3Q_1 - C_1Q_1}{C_2 - C_3} = 0,42m^3/s$. And therefore $Q_3 = Q_1 + Q_2 = 3,32m^3/s$.

$C_3Q_3 + C_4Q_4 = C_5(Q_4 + Q_3) \rightarrow Q_4 = \dfrac{C_5Q_3 - C_3Q_3}{C_4 - C_5} = 0,98m^3/s$. And therefore $Q_5 = Q_3 + Q_4 = 4,3m^3/s$.

Exercise 5.2.7

2007				2008			
H (cm)	Q(L/s)	ln(H(cm))	ln(Q(L/s))	H (cm)	Q(L/s)	ln(H(cm))	ln(Q(L/s))
43	29	3.76	3.37	59	65	4.08	4.17
33	7	3.5	1.95	34	0.5	3.53	-0.69
29	4	3.37	1.39	37	2	3.61	0.69
26	1	3.26	0	38	2	3.64	0.69
63	151	4.14	5.02	63	103.5	4.14	4.64
52	78	3.95	4.36	47	20	3.85	3





$H0_{2007} = 24,07cm$ and $H0_{2008} = 35,22cm$. R^2 does not permit the use of H0 over both years since it is lower than 0,8.

Exercise 5.3.1

a $A = 2km^2$, $A_{stream} = 1\%A$, $P = 5mm$

b $P = 10000m^3$

c $Q_{base} = 5L/s$

d $Q_{quick} = (0,5x0,5x20) + (2x0,5x20) = 18 + 72 = 90m^3$

e $\frac{Q_{quick}}{A} = Q_{quick,mm} = \frac{90}{2*10^6} = 4,5*10^{-5}m = 4,5*10^{-2}mm$

f $\frac{Q_{quick}}{P} = \frac{4,5*10^{-2}}{-5} = 0,9\%$

g $P_{river} = 100m^3$ (i.e. the volume that falls on the river itself), and therefore $P_{river} = 0,05mm$. $P_{river} \gtrapprox Q_{quick}$. This means that only rainfall on the river itself contributes significantly to quickflow.

Exercise 5.3.2

a $\%_{quick} = 4\% = \frac{Q_{quick}}{P} = \frac{10000}{250000}$

b & c $Q_{overland} = 0,5mm = 5*10^3m^3$

Exercise 5.4.1

a $Q_0 = 100m^3/s$ and $E^{-\alpha} = 0,9$. $\alpha = 4,0648*10^-8$ with t now in seconds. $S_{tot} = \frac{Q_0}{\alpha} = 2,460*10^9m^3$.

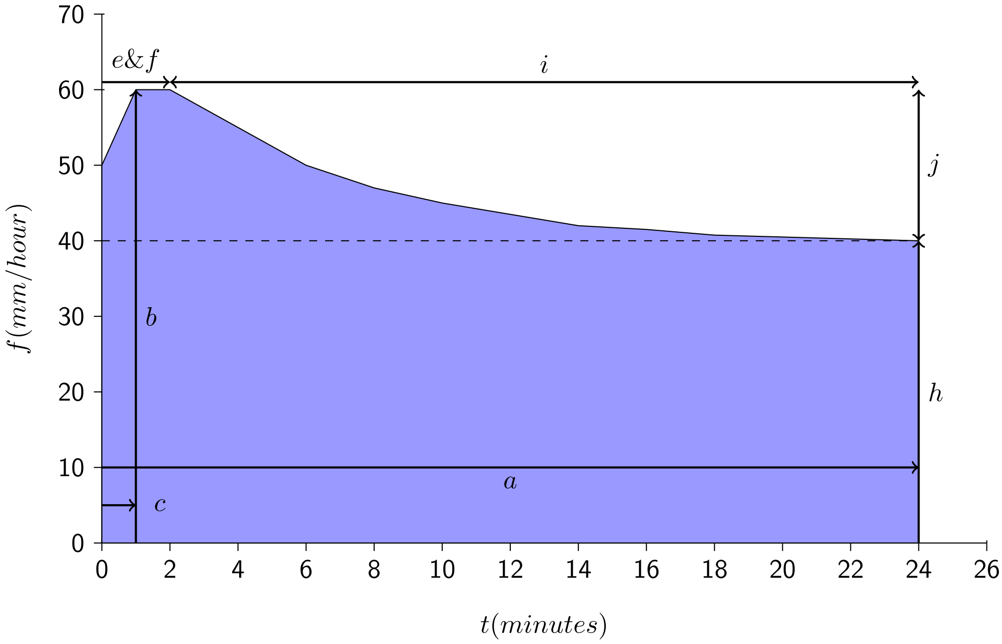
b $Q_{base1,5} = Q_0e^{-\alpha t} = 85,38m^3/s$ $Q_{base1,0} = Q_0e^{-\alpha t} = 90,00m^3/s$

c $S_{1,5} = \frac{Q_0e^{-\alpha t}}{\alpha} = 2,10*10^9m^3 = 85,47\%ofS_{tot}$

Exercise 5.4.2

$\frac{dQ}{\alpha} = -Qdt$
$\frac{dQ}{Q} = -\alpha dt$
$\int \frac{1}{Q}dQ = -\alpha t + C_1$
$\ln Q + C_2 = -\alpha t + C_1$
$Q + C_2 = e^{-\alpha t + C_1}$
$Q_t = e^{C_1}e^{-\alpha t}$ (with $C_2 = 0$)
$Q_t = Q_0e^{-\alpha t}$

Exercise 5.5



70
60
50
40
30
20
10
0
$f(mm/hour)$
$e\&f$
i
j
b
h
a
c
0 2 4 6 8 10 12 14 16 18 20 22 24 26
$t(minutes)$